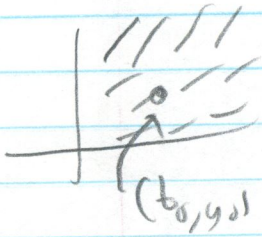
- PICK UP GRADED HW 2
- TEST PREP 2 (10 MIN END AT 10:40)
- SHOULD BE WORKING ON HW 2a AND EXAMINED.

MATH 307

HW 2a
Test Prep (10 minutes) → sol'ns hand out

2.7: Euler's Method



A numerical method for estimating values of sol'ns to

$$\frac{dy}{dt} = f(t,y) \qquad y(t_0) = y_0$$

IDEA ① $\frac{dy}{dt}$ = slope of tangent line

② TANGENT LINES LOOK LIKE $y = y_0 + m(t - t_0)$ (POINT; SLOPE OF THE TANGENT)

③ $\frac{dy}{dt} = f(t_0, y_0)$ = slope of tangent at (t_0, y_0)

Thus, even if we can't solve, we can find the tangent line.

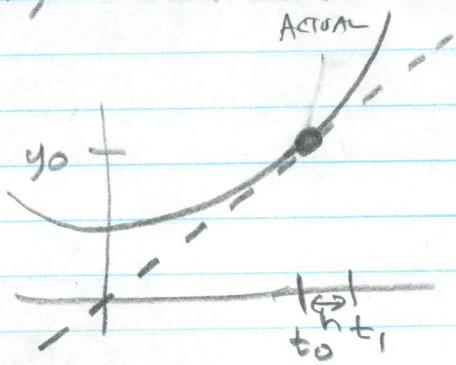
Let h = small number (step size)

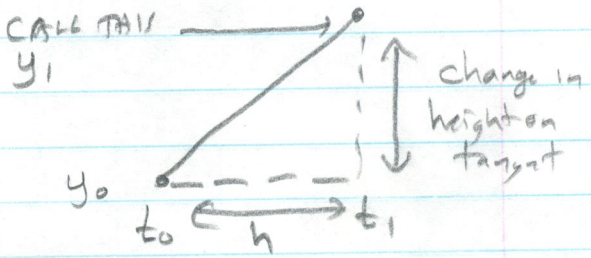
$t_1 = t_0 + h$

$y = y_0 + f(x_0, y_0)(t_1 - t_0)$

$y_1 = y_0 + f(x_0, y_0)\,h$

Euler's method





EULER'S METHOD

Given $\frac{dy}{dt} = f(t,y)$ AND $y(t_0) = y_0$.

0. Choose h
1. Compute $f(t_0, y_0)$
2. Calculate $t_1 = t_0 + h$
   $y_1 = y_0 + f(t_0, y_0)\,h$
3. Repeat steps 1 AND 2

$$t_{k+1} = t_k + h$$
$$y_{k+1} = y_k + f(t_k, y_k)\,h$$

Ex) $\frac{dy}{dt} = 2y - 1$ $\quad y(0) = 1$

$h = 0.25$

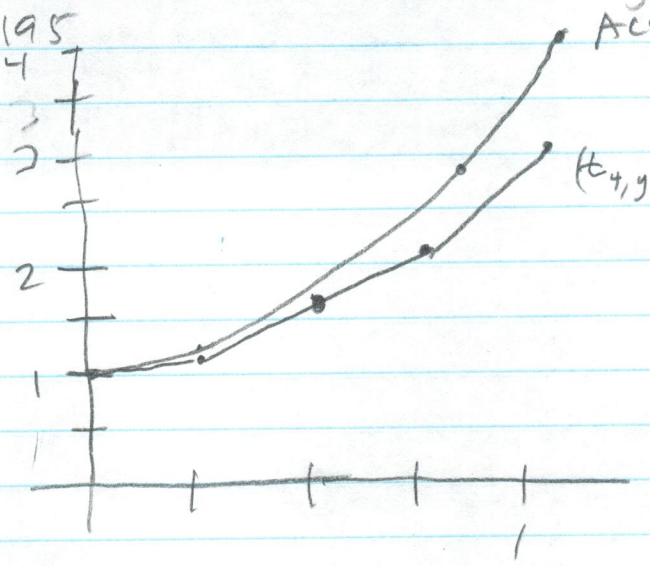
ASIDE: integrating factor
$\Rightarrow y(t) = \frac{1}{2}e^{2t} + \frac{1}{2}$

How?

STEP	t_k	y_k	$f(t_k, y_k)$	ACTUAL y
0	0	1	1	1
1	0.25	1.25	1.5	1.324
2	0.5	1.625	2.25	1.859
3	0.75	2.1875	3.375	2.741
4	1	3.03125	5.0625	4.195

Euler's method with $h = 0.25$ estimates $y(1) \approx 3.031$

ACTUAL ≈ 4.195



$\frac{dy}{dt} = 2y - 1 \quad y(0) = 1$

$h = 0.1$

STEP	t_n	y_n	$f(t_n, y_n)$
0	0	1	1
1	0.1	1.1	1.2
2	0.2	1.220	1.44
3	0.3	1.364	1.73
4	0.4	1.537	2.07
⋮	⋮		⋮
9	0.9	3.080	3.16
10	1.0	3.596	

ACTUAL 1.1

PREDICTS $y(1) \approx 3.596$

ACTUAL 4.195

$h = 0.01$

t_n	y_n
0	1
0.01	1.01
0.02	1.0202
⋮	⋮
0.99	4.051
1.00	4.122

$y(1) \approx 4.122$

ACTUAL 4.195

NOTES

- Smaller steps ⇒ better approx.
- farther away you go from initial condition ⇒ error tends to get worse
- if actual soln. is changing quickly (big changes in slopes) then approximation won't be as good.

You do $\frac{dy}{dt} = 2t - y \qquad y(2) = 4$

$h = 0.5$ ESTIMATE $y(4) = ?$

t_k	y_k	slope
2	4	$f(2,4) = 0$
2.5	4	$f(2.5,4) = 1$
3	4.5	$f(3,4.5) = 1.5$
3.5	5.25	$f(3.5,5.25) = 1.75$
4	6.125	

ACTUAL:

$y(t) = 2(t + e^{2-t} - 1)$

$y(4) \approx 6.270671$